DEPARTMENT OF COMMERCE
SINCLAIR WEEKS, Secretary

WEATHER BUREAU
F. W. REICHELDERFER, Chief

MONTHLY WEATHER REVIEW

JAMES E. CASKEY, JR., Editor

Volume 83
Number 9

SEPTEMBER 1955

Closed November 15, 1955
Issued December 15, 1955

AN INVESTIGATION OF CYCLONE DEVELOPMENT—STORM OF DECEMBER 13-15, 1951

LYNN L. MEANS [1]

U. S. Weather Bureau, Chicago, Ill.
in collaboration with the
Weather Forecasting Research Center,[2] Department of Meteorology, University of Chicago

[Manuscript received February 21, 1955; revised September 9, 1955]

ABSTRACT

The development of a storm which occurred in the United States during the period December 13-15, 1951 is investigated. An attempt is made to ascertain to what extent this development could be accounted for by the terms in the vorticity equation which derive from the vorticity advection and the thermal advection. It is found that the computed patterns agree well with those observed, although the numerical values are considerably exaggerated. Computed values of vertical velocity and divergence are compared with the observed patterns of clear sky and precipitation, and, on the whole, good agreement is found. The findings have gained further support from other storm studies and from experience in the use of vorticity charts and thermal advection charts in routine forecasting.

CONTENTS

1. INTRODUCTION

The present investigation forms a part of a program for studying cyclone development in the lower troposphere and deals with the fourth storm (Dec. 13-15, 1951) in this series chosen for a detailed fact-finding diagnostic study of development. The program which has been conducted jointly by the Weather Forecasting Research Center and the U. S. Weather Bureau District Forecast Center, Chicago, has been under the general direction of Prof. S. Petterssen. The general approach to and the theory underlying these investigations have been described in earlier papers (Petterssen [1], and Petterssen, Dunn, and Means [2]). For the convenience of the reader in interpreting terminology, symbols, and basic equations to be used in the sections that follow, these items from Petterssen's [1] theory of development are reproduced in the Appendix. In accordance with the general program, attempts have been made to compute vertical velocities, divergence, thermal advection, and vorticity changes. For a description of the methods of computation and the accuracies involved reference is made to an appendix to a report by Petterssen and Bradbury [3].

[1] Present address: National Weather Analysis Center, U. S. Weather Bureau, Washington, D. C.

[2] The research reported in this paper has been sponsored by the Geophysics Research Directorate of the U. S. Air Force Cambridge Research Center under contract No. AF 19(604)-309. It is published for technical information only, and does not necessarily represent recommendations or conclusions of the sponsoring agency.

2. GENERAL SYNOPSIS

The storm of December 13–15, 1951, was one of those moderate winter developments which in retrospect appear fairly routine, but which (as the writer can personally testify) offered real problems to the forecaster with reference to formation, movement, and increase in intensity, and also in related forecasts of precipitation, temperature, and wind. The following brief narrative of synoptic events during the development of this storm will provide the reader with some background for the more detailed analysis that follows. Charts illustrating the synoptic sequence are given in figures 1–3.

On December 12, 1951, a sea level pressure trough lay along the eastern slopes of the Rockies from Montana to western Texas (fig. 1a). On the 13th (fig. 1b) a center of low pressure was gradually organized in the northeastern corner of New Mexico as a fresh surge of polar air pushed south-southeastward along the eastern slopes through portions of Montana and Wyoming into Colorado. By 0300 GMT on the 14th (fig. 2a) the Low had begun to move eastward.

As the Low moved toward the lower Mississippi Valley a precipitation shield gradually developed and soon extended as far as 600 miles north and east from the center. During the day on the 14th a narrow band of heavy snow formed about 300 miles north of the low center, stretching from southeastern Nebraska across southern Iowa and extreme northern Illinois into southern Lower Michigan. Rainfall amounts of ¼ to ½ inch in 6 hours developed early on the 14th over Arkansas. The forward edge of this area of moderate to heavy rainfall moved rapidly up the Ohio Valley staying about 12 hours distance ahead of the movement of the sea level Low.

The circulation around the low center intensified considerably, especially after 1500 GMT, December 14 (fig. 2b) although the central pressure had dropped a maximum of only 10 mb. from 0300 GMT, December 14 to 0300 GMT, December 15 (fig. 3a). However this intensification did not continue long. As the Low moved into Pennsylvania on the 15th, a secondary wave formed off the New Jersey coast. Such a secondary formation is not infrequent. This secondary wave became the primary center by 1500 GMT December 15 (fig. 3b) as the cyclonic circulations merged off the New England coast.

At 500 mb. the broadscale pattern for the same synoptic sequence featured at 1500 GMT, December 12, a major trough over eastern Canada and the eastern United States, and a major ridge off the Pacific coast. This large amplitude ridge extended northwestward to the Bering Sea with strong winds and sharp anticyclonic curvature at the crest of the ridge. An important area of height falls formed in the current downstream from this ridge. The movement southeastward of this area of height falls into the col separating the Low off the southern California coast (fig. 1a) from the main current of the westerlies, opened up the closed Low so that by 1500 GMT on the 13th (fig. 1b) it was rapidly becoming converted into an open trough.

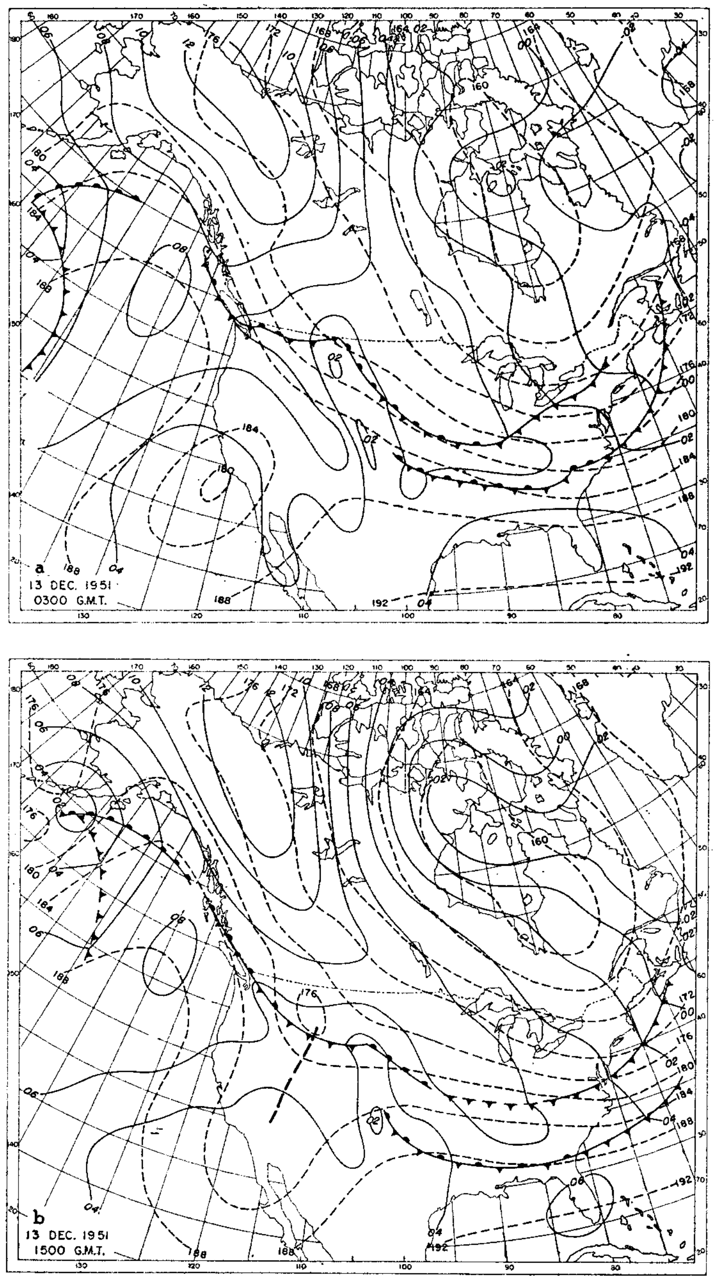

FIGURE 1.—Contours of 1000-mb. (solid lines) and 500-mb. (dashed lines) surfaces in hundreds of feet, and sea level fronts at 0300 and 1500 GMT, December 13, 1951.

The increase in amplitude of the trough downstream from the large amplitude ridge had now altered the picture over the southwestern United States considerably. On December 13 at 1500 GMT (fig. 1b) there was a minor trough over the northern Rockies as well as the nearly open trough just off the California coast which together, as we shall see later, formed a single area of positive vorticity advection at the 300-mb. level from Wyoming southward to New Mexico. It was at this time that the surface Low was becoming well-defined.

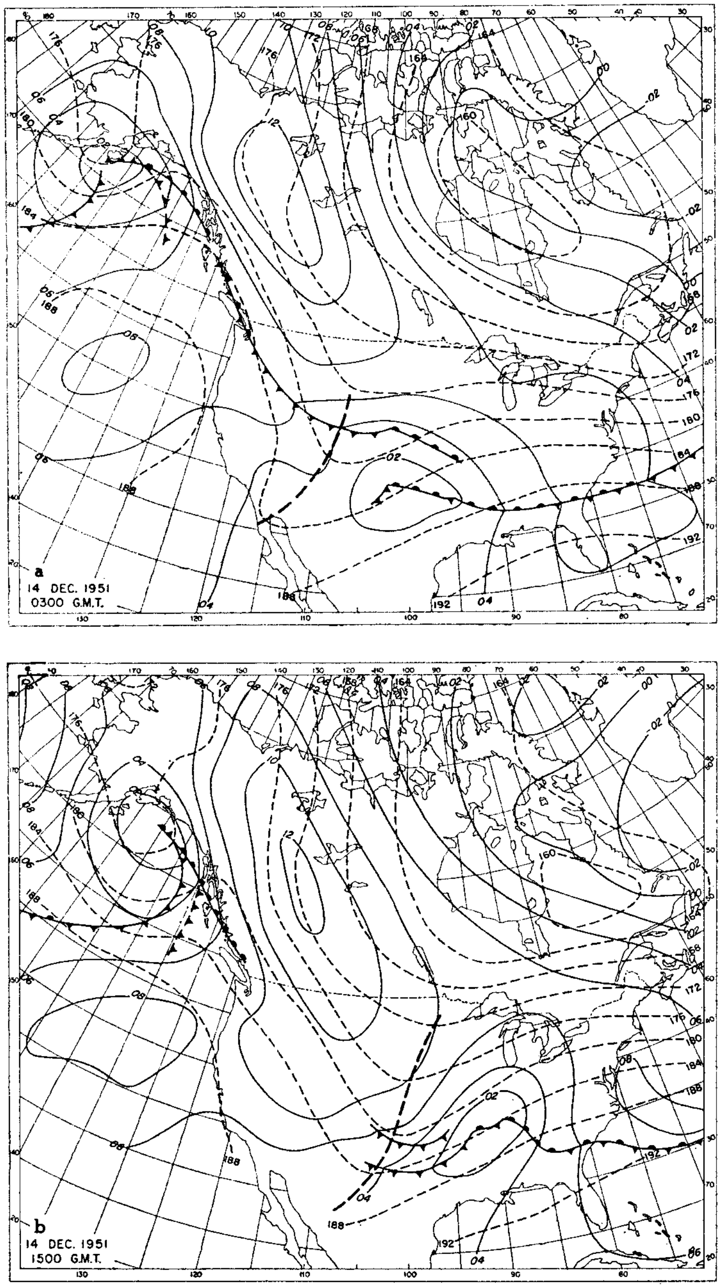

FIGURE 2.—Contours of 1000-mb. (solid lines) and 500-mb. (dashed lines) surfaces in hundreds of feet and sea level fronts at 0300 and 1500 GMT, December 14, 1951.

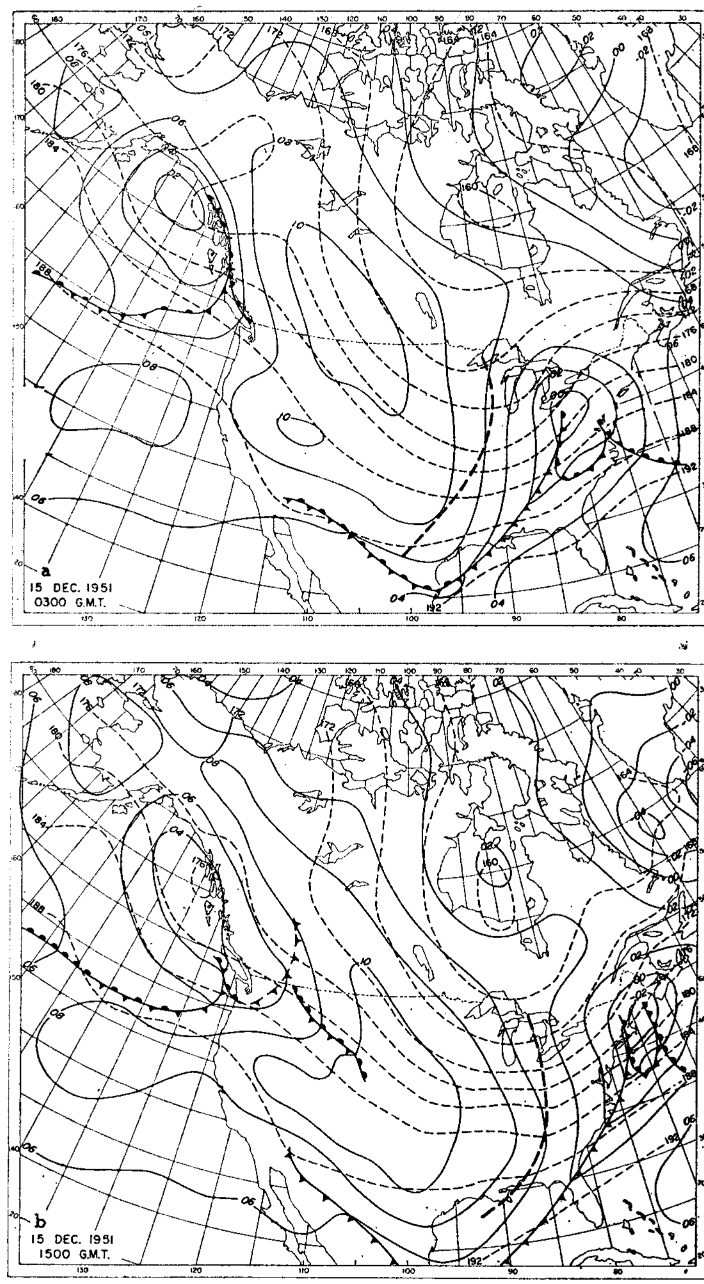

FIGURE 3.—Contours of 1000-mb. (solid lines) and 500-mb. (dashed lines) surfaces in hundreds of feet, and sea level fronts at 0300 and 1500 GMT, December 15, 1951.

By December 14, 0300 GMT (fig. 2a) the 500-mb. short wave pattern over the Rocky Mountain and Plateau area, as represented by the contours and isallohypses, was consolidated into a single wave which was obviously moving eastward in the main westerly current. Meanwhile strong rises in the contour heights were filling the trough over the eastern United States, and the broad flow pattern was tipping from mainly northwesterly flow over the central United States at 0300 GMT December 13 to mainly southwesterly at 1500 GMT December 14 (fig. 2b). The surface Low intensified under this southwesterly flow aloft.

The 500-mb. trough also intensified as it moved eastward across the United States. This intensification is consistent with the northeast to southwest tilt (fig. 2a) of the trough (Petterssen [4]) although other processes may have contributed also.

Computations by the aid of Petterssen's wave formula (Petterssen [4]) for 24-hour movements of the trough at the 500-mb. level are given in table 1.

By 0300 GMT, December 16, the major trough was again found over the Atlantic Coast States as it had been before this storm developed, and the main current at 500 mb.

TABLE 1.—*Computed 24-hour movement of the trough at the 500-mb. level*

Time		Computed ° Lat/day	Observed ° Lat/day
Date	*GMT*		
Dec. 13	1500	10	10
14	0300	11	13
14	1500	11	13

had again reversed its tilt over the Central States and was somewhat similar to that of 0300 GMT, December 13 (fig. 1a).

It was apparent that a long wave, or a mean trough, was present over the United States during the period of this development, but owing to superimposition of minor waves, its exact location and sequence appeared rather indefinite on the twice-daily charts. During the first half of December each of several minor waves was vigorous and tilted the 500-mb. flow pattern over the central United States first to southwest-to-northeast flow, then to northwest-to-southeast flow. Each wave moved through in such rapid order that one could hardly regard the entire sequence as repeated retrogression and progression of the long wave pattern.

3. TEST OF A SIMPLIFIED HYPOTHESIS

In the study of this situation the applicability of a simple working hypothesis will first be analyzed, and a more complete discussion of the development process will be given later.

The simple working hypothesis to be tested is as follows (Petterssen, Dunn and Means [2]): *The establishment of a region of appreciable low-level convergence results when and where an area of appreciable positive vorticity advection in the middle and upper troposphere becomes superimposed upon a low-level frontal system.*

Various arguments can be offered in support of this simplification of Petterssen's [1] theory of development. For example, initially, before any appreciable circulatory motion has been created, the sea level vorticity advection, the Laplacian of the thermal advection, and the vertical velocity-stability terms are likely to be small in the vicinity of the center of the weak surface Low. Neglecting these terms and the non-adiabatic term, equation (2.5) (Appendix) reduces to $D_0Q_0 = -A_{QL}$ where A_{QL} is the vorticity advection at the level of non-divergence, and D_0 and Q_0 are the divergence and the absolute vorticity (vertical component) at sea level. Since Q_0 remains positive, A_{QL} gives a qualitative indication of convergence at sea level.

Since Petterssen's approach to the development problem deals with the level of nondivergence and the layer between that level and sea level, a primary consideration in the testing of this hypothesis is the height of the level of nondivergence. In typical cases the level of nondivergence is in the upper troposphere or near the tropopause before and during the early stages of development. The compensation in the divergence field is then primarily between the upper troposphere and stratosphere. Development becomes pronounced when the level of nondivergence drops so that the greater degree of compensation occurs within the troposphere (Petterssen [1] and Petterssen, Dunn and Means [2]).

The question with reference to this particular case then is whether, using the 300-mb. level as level of nondivergence, the superimposition of an area of appreciable positive vorticity advection at that level upon a low-level frontal system, was associated with the establishment of a region of appreciable low-level convergence.

Again examining the synoptic situation, at 0300 GMT, December 13 (fig. 4a), a short-wave trough was beginning to develop in the Far Northwest and the 300-mb. closed Low off the southern California coast was just beginning to open up. Two centers of positive vorticity advection were present at 300 mb. in association with these two systems. By 1500 GMT, December 13 (fig. 4b), the short wave was over the northern Rockies, and the closed Low aloft in the Southwest was a nearly open trough. The corresponding positive vorticity advection areas were joined into one which extended from North Dakota and Wyoming southward to New Mexico. The southernmost forward edge of this pattern was becoming superimposed upon the baroclinic zone associated with the quasi-stationary polar-tropical air front in the vicinity of the Texas Panhandle, and by 0300 GMT, December 14 (figs. 2a and 4c) a definite low center was present just north of Amarillo.

The Low moved eastward on December 14. However, the main body of the area of appreciable positive vorticity advection on the 300-mb. chart lagged behind the newly-developed Low as it moved eastward to northern Arkansas by 1500 GMT, December 14 (fig. 4d) and little further development of the sea level Low occurred up to this stage. While precipitation increased east and north of the Low, very little precipitation developed over the center of the Low, and the central pressure of the newly formed Low remained about the same at 1500 GMT, December 14.

The 500-mb. trough computation (see Section 2) at 0300 GMT, December 14 indicated that the trough should continue moving eastward at a moderate speed, possibly with some acceleration. The vorticity advection at the 500- and 300-mb. levels showed a very marked increase over the center of the Low between 1500 GMT on the 14th and 0300 GMT on the 15th (fig. 4d and e). This was associated with (1) a decrease in central pressure, and increased circulation around the Low (fig. 3a); (2) a lowering of the level of nondivergence (as will be shown later, fig. 6) from about 400 mb. to 600 mb.; and (3) a marked increase in the area and amounts of precipitation in the vicinity of the center of the Low. (All of these occurred between 1500 GMT on the 14th and 0300 GMT on the 15th.)

Vorticity advection amounts for 500 and 300 mb. were still strong but decreasing over the low center at 1500 GMT, December 15 (fig. 4f).

This case then showed general agreement with the simplified hypothesis and with the typical sequence of

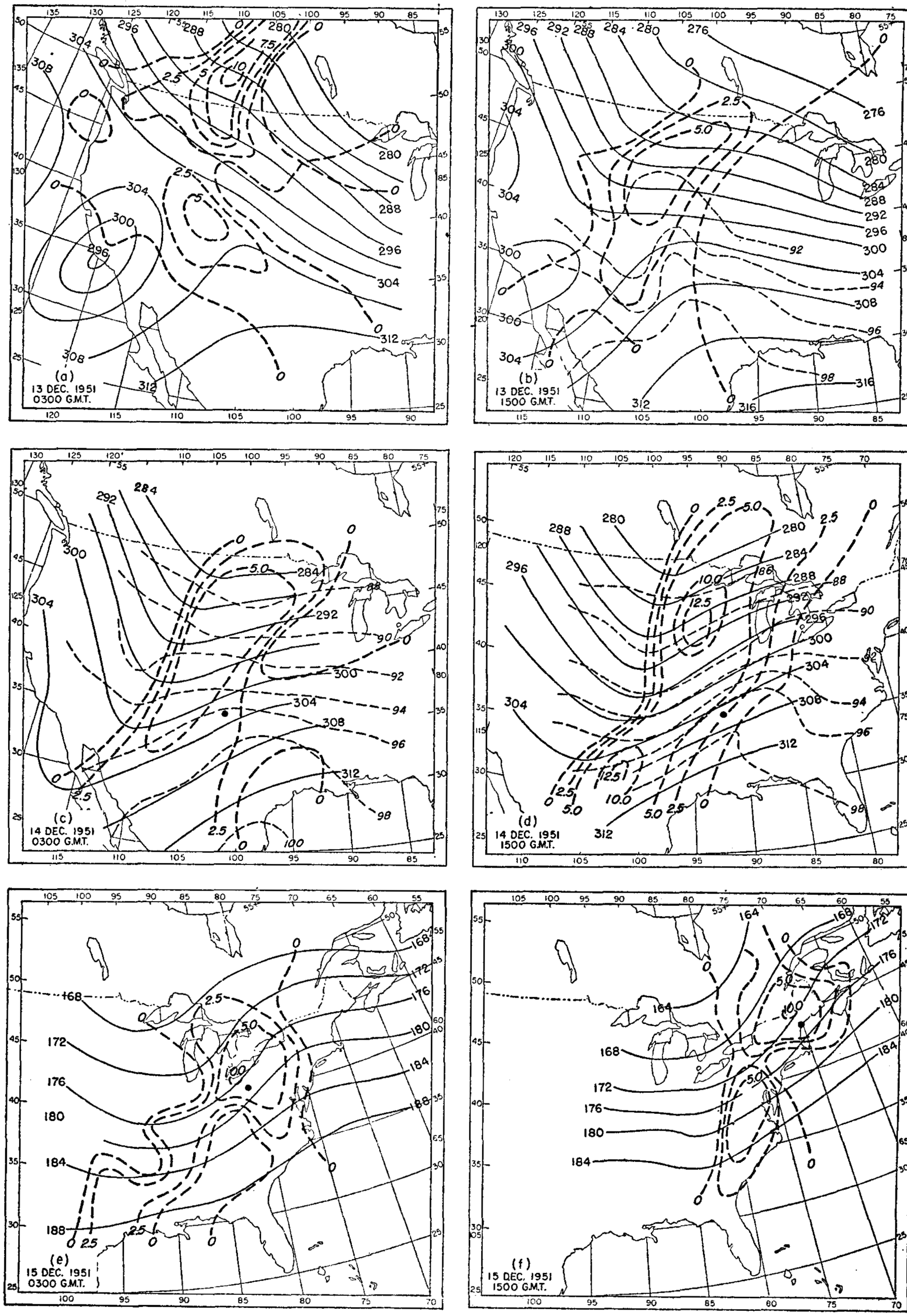

FIGURE 4.—(a-d) Contours of 300-mb. surface (solid lines) and thickness 1000–700 mb. (light broken lines) in hundreds of feet, and vorticity advection (heavy broken lines) at 300 mb. in units of 10^{-9} sec.$^{-2}$. (c-f) Dot represents surface center. (e-f) 500-mb. contours (solid lines) and vorticity advection (broken lines) 0300 and 1500 GMT, December 13, 14, 15, 1951.

events observed in a number of other developments. The Low formed when the forward edge of the positive vorticity advection in the upper troposphere moved over the low-level baroclinic zone associated with the quasi-stationary front. Precipitation increased, circulation about the Low intensified together with a decrease in central pressure of the Low, and the level of nondivergence dropped as the area of appreciable vorticity advection in the upper troposphere became established over the surface low center.

4. COMPARISON OF CONTRIBUTIONS BY VORTICITY ADVECTION AND THICKNESS ADVECTION

Since the Laplacian of the vertical motion-stability term and nonadiabatic term (see equation (2.6), Appendix) are not readily obtainable from synoptic analyses, an attempt was made to compare the general features of the observed development with the contributions of the vorticity advection and Laplacian of the thermal advection terms. The contributions of the vorticity advection and the thermal advection terms will be discussed with reference to (1) the center of the moving Low, and (2) the environment of the low center.

Computation of thermal and vorticity advection terms *over the center* of the Low gave values which tended to increase with time (see table 2).

The vorticity advection term was positive and contributed to a greater degree than the thermal term to low-level convergence throughout the period. The Laplacian of thermal advection term on the 14th at 0300 GMT contributed to weak low-level divergence. On the 14th at 1500 GMT it contributed to weak low-level convergence. On the 15th at both 0300 and 1500 GMT the thermal advection term contributed significantly to low level convergence, but the verticity advection increased even more. Between the 14th at 1500 GMT and the 15th at 0300 GMT when the Low showed rapid development, the level of nondivergence dropped from about the 400-mb. level to about 600 mb. (fig. 6). Therefore the 500–300-mb. layer was not considered for 0300 and 1500 GMT on the 15th as this was above the level of nondivergence. It is apparent from the data that the contribution of the Laplacian of thermal advection term was very small over the center of the Low until the Low acquired considerable intensity; then this term contributed appreciably to the further intensification.

TABLE 2.—*Comparison of contributions by thermal advection and vorticity advection. Units 10^{-9} sec.$^{-2}$*

Time		$-\frac{g}{f}\nabla^2 A_h$			Sum for layer	A_{QL}		Total
		1000–700 mb.	700–500 mb.	500–300 mb.		500 mb.	300 mb.	
Date	*GMT*							
Dec. 14	0300	−.3	−1.8	1.7	−0.4		3.5	3.1
14	1500	−.1	1.6	−0.9	.6		2.0	2.6
15	0300	.8	3.3		4.1	8.0		12.1
15	1500	3.8	1.6		5.4	7.0		12.4

A somewhat different comparison of the contributions of the thermal and vorticity terms toward development may be found by examining those terms together with observed $\frac{\partial Q_0}{\partial t}$ data for grid points surrounding the low center, all within a radius of 700 km. Twelve grid points were taken around the center of the Low for 0300 and 1500 GMT on the 14th, and 0300 GMT on the 15th. Only nine points were used at 1500 GMT on the 15th due to lack of reliable data off the East Coast. The relationship among these terms is given by equation (2.7) (Appendix):

$$\frac{\partial Q_0}{\partial t}=A_{QL}-\frac{g}{f}\nabla^2 A_h$$

but $\frac{\partial Q_0}{\partial t}$ can be evaluated by differentiating $Q_0=\frac{g}{f}\nabla^2 z+f$ where z is the height of the 1000-mb. level. Thus

$$\frac{\partial Q_0}{\partial t}=\frac{g}{f}\nabla^2\frac{\partial z}{\partial t}$$

The average local rate of change of vorticity with time near the surface then can be evaluated using sea level pressure changes or 1000-mb. height changes. This was done using 12-hour sea level pressure changes. The Laplacian of the tendency field was evaluated from a grid similar to that used in vorticity computations, and isopleths were drawn giving patterns of observed $\frac{\partial Q_0}{\partial t}$. A qualitative comparison of these patterns with patterns of $\frac{\partial Q_0}{\partial t}$ computed from the Laplacian of thermal advection and vorticity advection terms will be given later in this section.

In order to provide a more quantitative comparison of these terms a regression equation and correlation coefficients were computed for the relationship of $\frac{\partial Q_0}{\partial t}$ (from pressure changes) to $\left(A_{QL}-\frac{g}{f}\nabla^2 A_h\right)$. Although the center of the 12-hour time period over which the pressure changes were averaged was 3 hours later than the time for which the instantaneous vorticity and thermal advection terms were computed, this was partially allowed for by positioning the grid points similarly with reference to the center of the Low in each case. Data then were extracted for the grid points and the regression computed, giving:

$$\frac{\partial Q_0}{\partial t}=.3756\,(10^{-9}\text{ sec}^{-2})+.2012\left(-\frac{g}{f}\nabla^2 A_h\right)-.0144\,(A_{QL})$$

The multiple correlation coefficient was .74, with its square, .54, indicating that more than half of the variability in the observed local time changes in vorticity was accounted for. The correlation of the local time change of vorticity with the Laplacian of thermal advection term alone was .736 and the correlation with the vorticity advection term alone was .24, and partial correlation coefficients were .72 and .02.

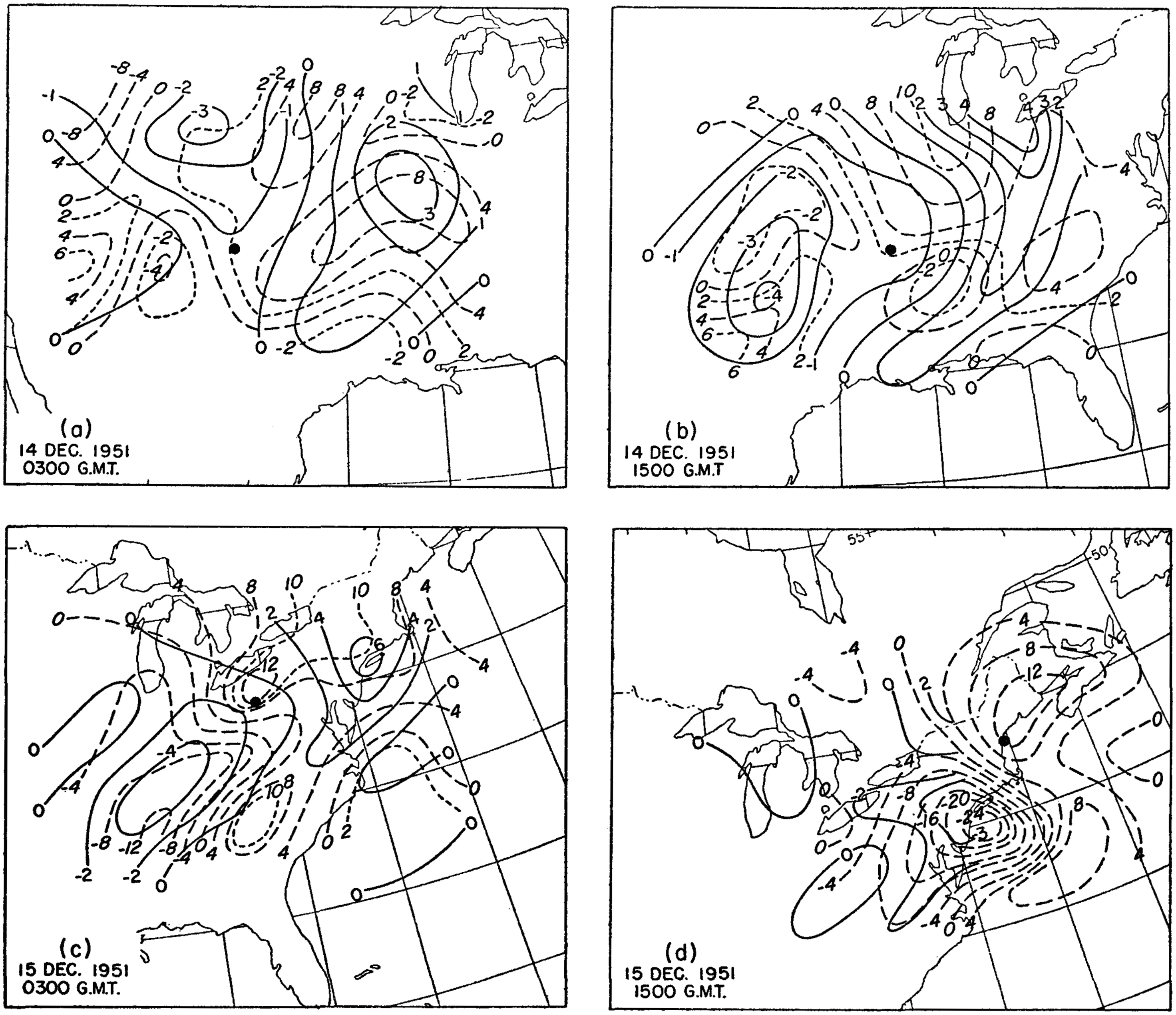

FIGURE 5.—Observed (solid lines) and computed (broken lines) patterns of local rate of change of 1000-mb. vorticity in units of 10^{-9} sec.$^{-2}$. Large dot indicates position of surface center.

The above correlation coefficients suggest that in the environment of the Low, but not directly over the center, the Laplacian of the thermal advection appears to be more important in this case than the vorticity advection term. This contrasts with the situation over the center of the Low where it appears that in this synoptic situation the vorticity advection term was the primary factor. The Laplacian of the thermal advection term was rather large when the Low first began moving across the Texas Panhandle, but the significant amounts contributing to low-level divergence were to the rear of the Low, and significant amounts contributing to low-level convergence were in advance of the Low. These significant values of the Laplacian of thermal advection term straddled the center giving little net contribution in the vicinity of the center. If **C** is the velocity of the sea level system one may write $\frac{\partial Q_0}{\partial t}=\frac{\delta Q_0}{\delta t}-\mathbf{C}\cdot\nabla Q_0$ where $\frac{\delta Q_0}{\delta t}$ is the local rate of intensification in the moving coordinate system. (See Appendix, section 2.) Since the local change is made up of both the intensification and convective terms, and since only moderate intensification occurred in this storm, the local changes largely reflected the movement ($\mathbf{C}\cdot\nabla Q_0$) of the Low. Since the Laplacian of the thermal advection term was predominant for areas away from the center as indicated by the regression and correlation coefficients, this term probably was associated mostly with movement of the Low.

Since there is at least some suggestion of association in this case of central development with vorticity advection (table 2), and of movement with Laplacian of thermal advection (above regression equation) the regression constants given above may vary considerably from one case to another depending upon the proportion of the local change of vorticity with time that is due to development, as compared with the proportion that is due to movement of the system. Also the constants may vary from one time period to another during the development of a given storm. However, there are insufficient data in this one case to ascertain with any degree of reliability to what degree this is true.

In general, we probably should not attempt to regard the various terms in equation (2.6) (Appendix) as being entirely independent factors, but rather as a complex process of interrelated factors.

Going back now to the observed values of $\frac{\partial Q_0}{\partial t}$ as derived from $\frac{g}{f}\nabla^2\frac{\partial z}{\partial t}$ we compare analyzed patterns of this factor with analyzed patterns of $\frac{\partial Q_0}{\partial t}$ as computed from vorticity advection and Laplacian of thermal advection terms. The qualitative similarities of the two patterns of local changes of vorticity with time are easily seen in figures 5a, b, c, d. The agreement was better after the cyclone had intensified than before, especially at 0300 GMT on the 15th (fig. 5c).

Quantitatively, the local positive vorticity change values computed from vorticity and thermal advection were three to four times as great as those from the observed tendencies. Negative values were exaggerated somewhat more. The exaggeration of the computed tendencies is due to several factors. For one, the computed tendencies are larger due to surface elevation. Since much of the topography is above the 1000-mb. level, the computed thermal advection term for the layer 1000–700 mb. tends to be too large. On the other hand, nonadiabatic effects tend to counteract thermal advection. The vertical motion-stability, $\omega(\Gamma_a-\Gamma)$ term which was also neglected, usually tends to act as a brake except, of course, when $\Gamma>\Gamma_a$, which may occur briefly over somewhat limited portions of the cyclonic system. Therefore omission of this term could cause computed changes to be too large. An appreciable portion of the error may be due to the geostrophic assumption and to frictional forces which would also cause the computed changes to be too large. Computed negative values were especially exaggerated as compared with observed. This is what we would expect since ascending motion is largely wet-adiabatic while downward motion is largely dry-adiabatic. The omission of the buoyancy term, then, tends to exaggerate the computed *negative* vorticity changes more than the positive vorticity changes.

In general the vorticity transport terms were greater above 500 mb. and the Laplacian of thermal advection terms were greater below 500 mb.

To summarize the results of this section it may be stated that general qualitative agreement is found between the values of $\frac{\partial Q_0}{\partial t}$ derived from the observed tendency field and the values derived from computations of the Laplacian of thermal advection and vorticity advection terms. Although the computed patterns differ quantitatively by a factor of 4 or 5, the square of the multiple correlation coefficient suggests that about half the variation in the observed local change is accounted for by variations in the thermal advection and vorticity advection terms. The other half of the variation in the observed values is unexplained due to the dropping of buoyancy and nonadiabatic terms, due to assumptions of frictionless geostrophic motion, due to substitution of finite increments of time and space in place of the infinitesimals of the basic equations, and due to errors of observations, analysis, and evaluations.

5. FURTHER DISCUSSION ON OBSERVED DEVELOPMENT

The observed development will be discussed further, making use of the following data: (1) time cross-sections of vertical motion, vorticity, and divergence over the center of the sea level Low, (2) the development and spread of precipitation, and (3) increase in intensity of the circulation and decrease in central pressure of the Low.

(1) An indication of development over the low center is given by time cross-sections of vorticity, vertical motions, and divergence over the center of the Low in figure 6. Vorticities over the center of the Low were interpolated from vorticity charts for constant pressure surfaces. The method for computing the vertical velocities is given in the Appendix, section 3. The divergence was computed from the vertical velocity field using the continuity equation, Div $\mathbf{V}=-\frac{\partial\omega}{\partial p}$.

The major development occurred from 1500 GMT on the 14th to 0300 GMT on the 15th (fig. 6). Upward velocities and low-level convergence were greatest at 1500 GMT on the 14th. The level of nondivergence, as indicated by the heavy solid line, dropped rapidly during that period from about the 400-mb. level to about the 600-mb. level at 0300 GMT on the 15th. The sudden "flareup" of development was preceded by weak upward motion at the 850-mb. level at 0300 GMT on the 14th when the Low was just beginning to move away from its place of formation. Downward motion, which was especially marked at the 300- and 500-mb. levels at this time, was probably associated with flow across the mountains which were close by to the west. Following the flareup mean downward motion developed at 1500 GMT on the 15th throughout the air column over the low center, although some low-level convergence persisted. These patterns were weak in the lower troposphere, but divergence and downward motion were strong in the upper troposphere. This was consistent with the fact that by this time the original Low was beginning to fill

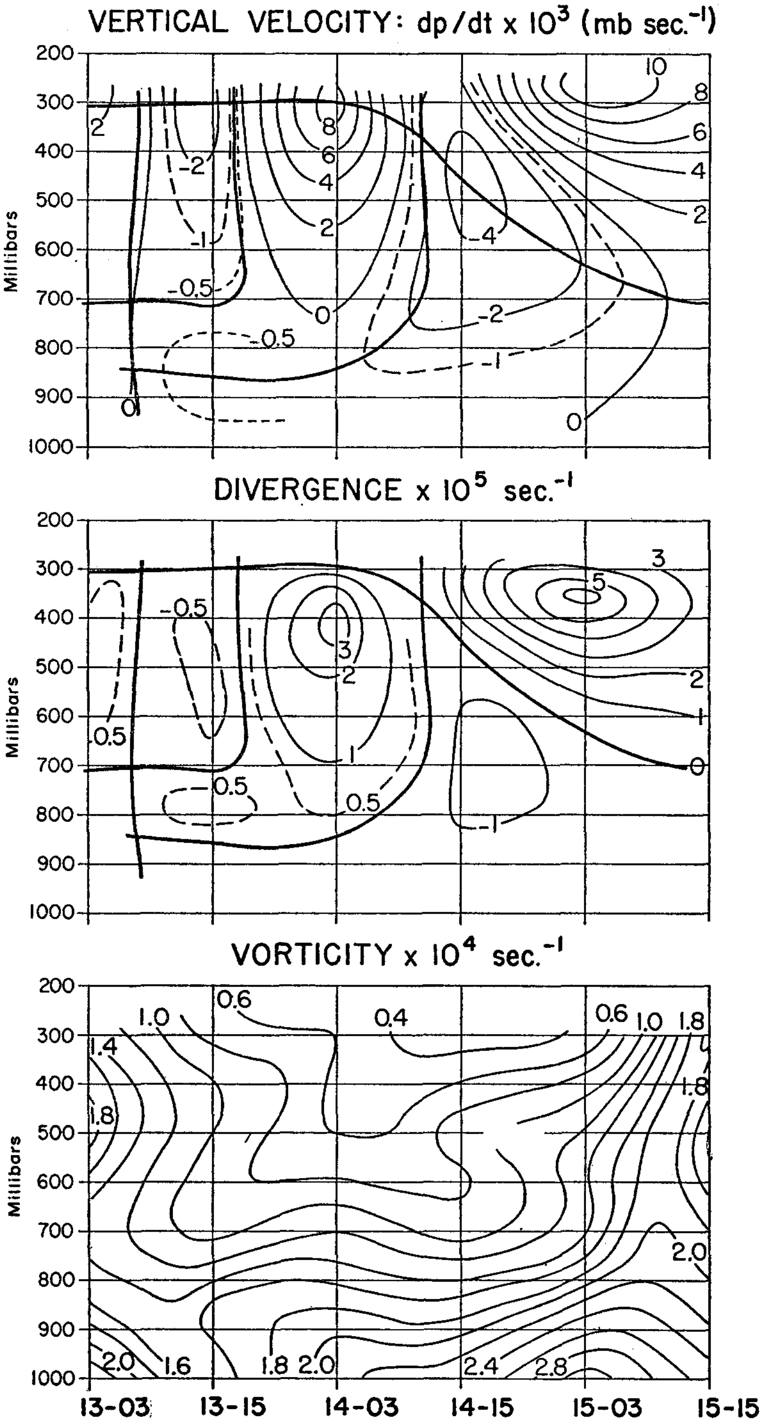

FIGURE 6.—Distribution of absolute vorticity, divergence, and vertical velocity in the column over the sea level center for the period December 13–15, 1951.

as a deepening secondary wave off the coast of Maine became the principal center. This was the secondary that had been forming off the New Jersey coast as the original Low moved into Pennsylvania.

These vertical motions and indications of convergence and divergence (fig. 6) over the center of the Low were consistent with the occurrence of precipitation at the center of the Low. No significant precipitation occurred directly at the center before 1500 GMT on the 14th although significant amounts occurred to the north and east. But shortly after 1500 GMT on the 14th, when considerable upward motion and low-level convergence were indicated, precipitation was moderate to heavy at the low center. This continued into the 15th. By 1500 GMT on the 15th when mean downward motions were weakly indicated in the lower levels, precipitation amounts in the vicinity of the low center were less than 1/10 inch in 6 hours.

In the time cross-section (fig. 6) vorticities associated with the moving impulse apparently decreased as it moved south-southeastward from Montana along the eastern slopes. But with the consolidation of the contributions from two vorticity advection areas, one with the short wave over the northern Rockies and the other with the opening and moving out of the cyclonic system in the Southwest, some increase in low-level vorticity was associated with organization of the closed sea level low center over northeastern New Mexico between 1500 GMT on the 13th and 0300 GMT on the 14th. Little further development occurred between 0300 and 1500 GMT on the 14th. The most important increase, between 1500 GMT on the 14th and 0300 GMT on the 15th was consistent with other indications of development as discussed earlier. The increase in vorticity directly over the 1000-mb. low center lagged behind the increase at 1000 mb. By 1500 GMT on the 15th vorticities at 300 mb. were reaching a peak value while 1000-mb. vorticities were beginning to decrease.

(2) A comparison of 6-hourly precipitation patterns with computed $\frac{\partial Q_0}{\partial t}$ patterns as determined from the vorticity advection and the Laplacian of the thermal advection, is of some interest. The 6-hourly precipitation period bracketed the time at which the computations were made, e. g., 1230–1830 GMT precipitation accumulations correspond to 1500 GMT computations of $\frac{\partial Q_0}{\partial t}$. (See fig. 7a, b, c, d). Positive values of $\frac{\partial Q_0}{\partial t}$ give at least a qualitative indication of low-level convergence, and negative values, low-level divergence. Two categories of 6-hourly precipitation are outlined: less than .25 inch, and .25 inch and more. Smaller amounts of precipitation, usually less than .10 inch, were frequently associated with orographic effects or low-level instability and stratocumulus clouds where cold air was flowing over warmer ground or over the warmer waters of the Great Lakes, and some of these small precipitation amounts occurred locally despite indications of computed low-level divergence. Areas of clear skies or scattered to broken low clouds are also indicated in figure 7 and may be compared with areas of computed low-level divergence.

Some of the precipitation indicated at 0300 GMT on the 14th (fig. 7a) along the eastern slopes of the Rockies was undoubtedly associated with orographic lifting in the east to northeast winds at low levels over that area especially since it ended rapidly as winds shifted to a downslope

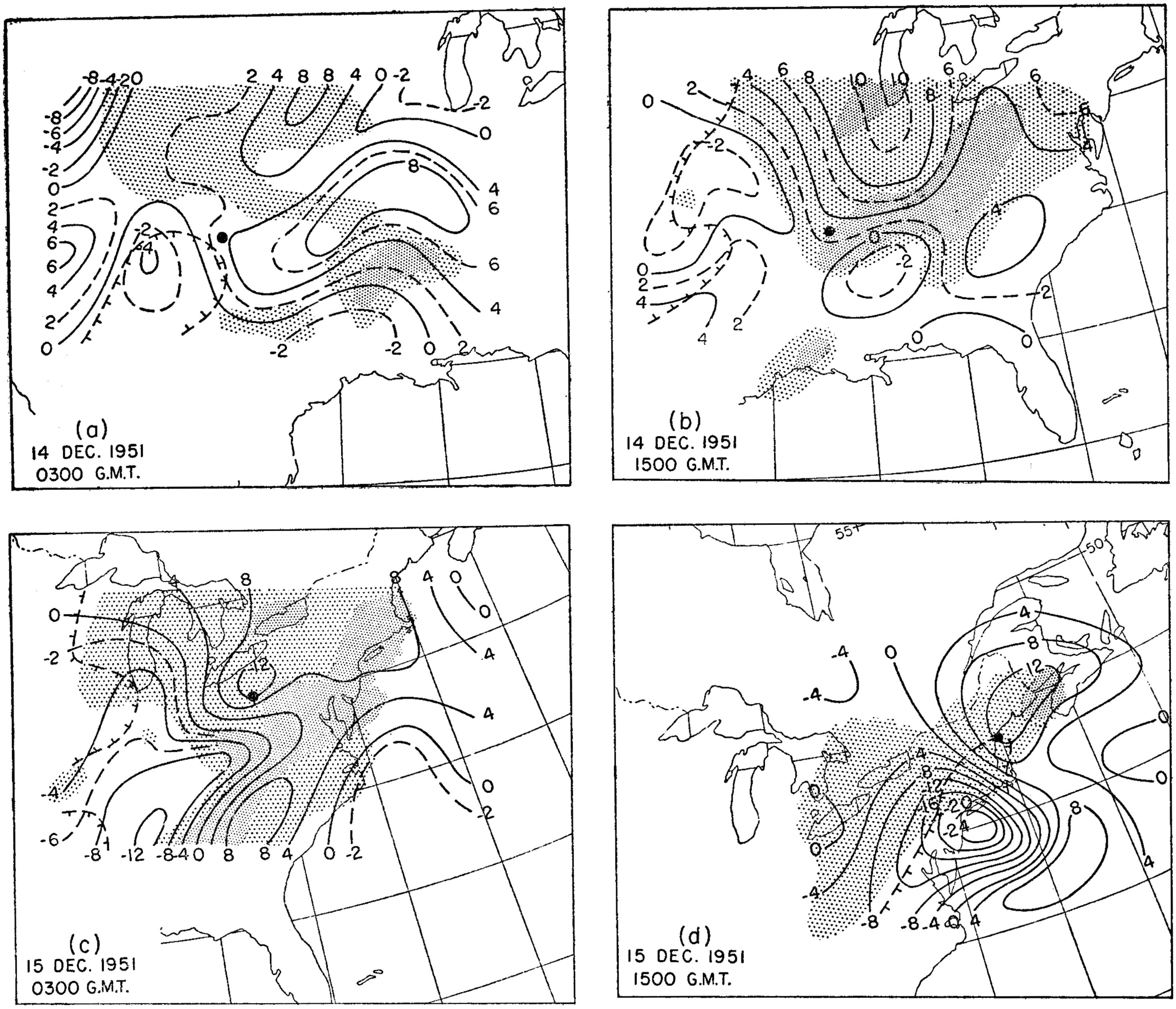

FIGURE 7.—Comparison of 6-hour precipitation amounts with areas of computed low-level convergence. Light shading represents amount less than 0.25 inch and heavy shading, amounts equal to or greater than 0.25 inch. ⊥⊥⊥ line encloses areas with clearing skies. Large dot indicates position of surface center.

component. Really significant amounts of rainfall were developing at this time over Arkansas along the southern edge of a computed area of low-level convergence. The heavier amounts of precipitation continued to be associated through the period of this development with the southern portion of the area of low-level convergence. Warmer and more moist air was present in the lower layers in that portion of the convergence area. This emphasizes the importance of moisture content of the air and probably also its degree of stability.

The Laplacian of the thermal advection amounts were larger in the area ahead of the Low than at the low center. They were also usually larger than the vorticity advection amounts in the precipitation area north and east of the storm center.

An interesting feature of the chart for 1500 GMT, December 14 (fig. 7b) is the isolated area of heavier precipitation southwest of the southern tip of Lake Michigan, which suggests that the flow of cold air in the east to northeast surface winds across the relatively warm Lake waters contributed locally to increased precipitation amounts. An accumulation of 10 inches of snow was reported at Chicago.

The layer 1000–300 mb. is used for comparison at 0300 and 1500 GMT on the 14th while the layer 1000–500 mb. is used at 0300 and 1500 GMT on the 15th due to the fact

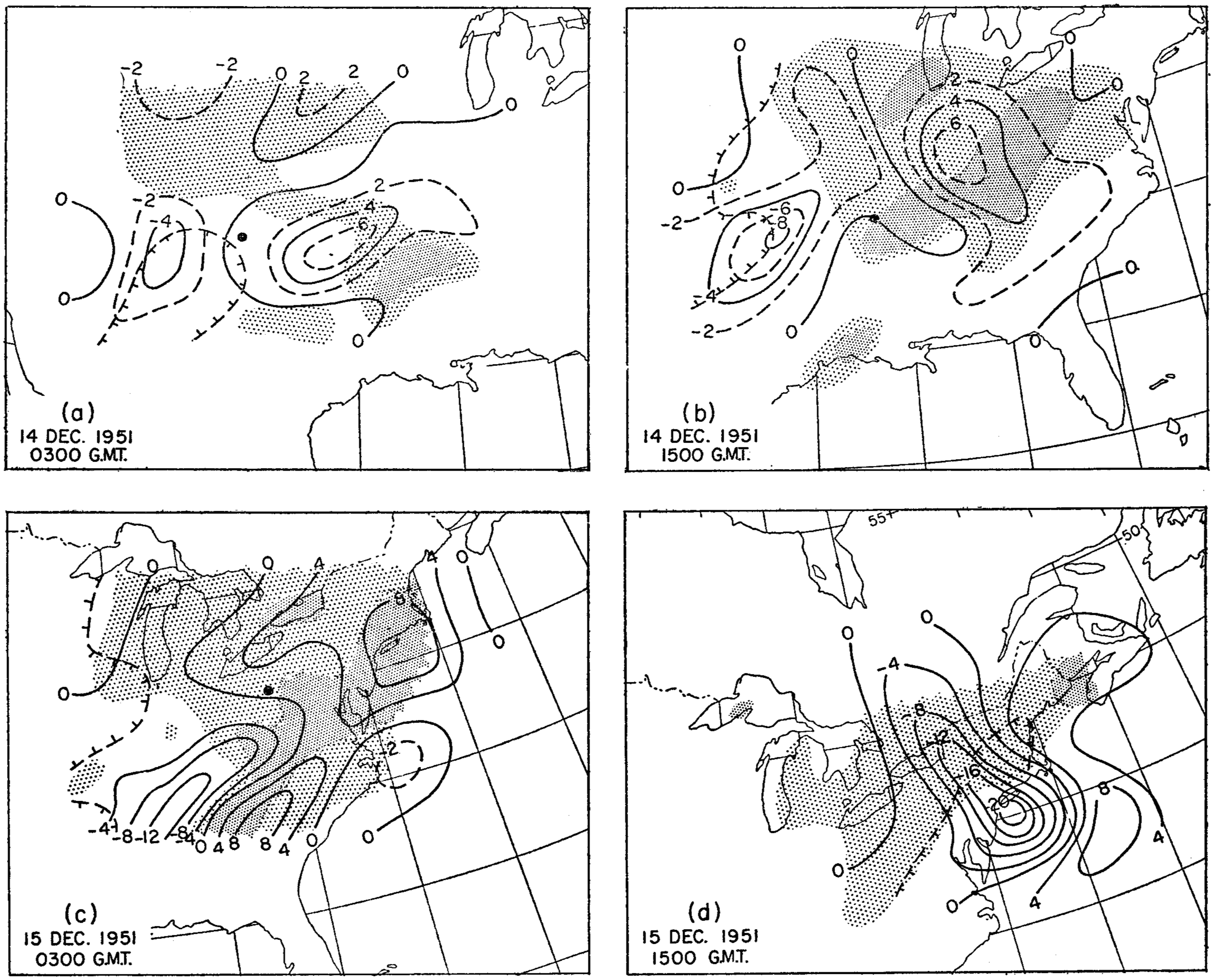

FIGURE 8.—Comparison of 6-hour precipitation amounts with areas of low-level convergence as computed from Laplacian of thermal advection term, 1000-700-mb. layer. Light shading represents amounts less than 0.25 inch and heavy shading, amounts equal to or greater than 0.25 inch. ⊥⊥⊥ line encloses areas with clearing skies. Large dot indicates position of surface center.

that the level of non-divergence dropped between 1500 GMT on the 14th and 0300 GMT on the 15th.

By 1500 GMT on the 15th much of the comparison is lost because the main body of precipitation was moving off the northeast coast. Considerable snow flurry activity persisted between the Great Lakes and the Appalachians as is usually the case to the rear of such winter storms. Areas of computed low-level divergence were in general qualitative agreement with areas of clear or clearing skies, although in some areas stratocumulus clouds and even snow flurries tended to persist for a while to the rear of the development due to orographic and air mass modification factors.

In the analysis of these data, precipitation patterns appeared to be closely related to the Laplacian of the thermal advection term, $\left(-\frac{g}{f}\nabla^2 A_h\right)$, especially that for the lower troposphere. (Appleby [5] has also found such a relation using forecast patterns of temperature advection at 850 mb.) Therefore a comparison was made between the 6-hourly precipitation and the Laplacian of the thermal advection patterns for the layer 1000–700 mb. This is shown in figure 8. General qualitative agreement is apparent in this case as was found in figure 7.

This was supported by a correlation between the simple $\left(-\frac{g}{f}\nabla^2 A_h\right)_{1000-700}$ and the more elaborate values $A_{QL}-\left(\frac{g}{f}\nabla^2 A_h\right)_{1000-300 \text{ or } 1000-500}$ upon which patterns in figure 8 and figure 7 are based respectively. Data were selected

TABLE 3.—*Sequence of central pressure and circulation in storm of December 13–15, 1951*

Time		Central pressure	Decrease	Circulation	Increase
Date	*GMT*	*mb.*			
Dec. 13	1230	1008		32	
			−8		10
14	0030	1000		42	
			1		1
14	0630	1001		43	
			0		1
14	1230	1001		44	
			0		−1
14	1830	1001		43	
			−4		15
15	0030	997		58	
			−5		−1
15	0630	992		57	
			0		
15	1230	992		[1]	

[1] No value computed at edge of chart.

at 48 grid points on *each* of the two sets of charts. The correlation coefficient was .81, thereby indicating that the simple parameter accounted for two-thirds of the variability in the more complete function.

This would suggest that the Laplacian of the thermal advection for the 1000–700-mb. layer gives a simple first approximation to the computed $\frac{\partial Q_0}{\partial t}$ as derived from the vorticity advection at the level of nondivergence together with the Laplacian of thermal advection for the entire layer 1000 mb. to the level of nondivergence.

(3). The development of the storm of December 13–15, 1951, has been discussed primarily with reference to vertical motions and associated precipitation, computed and observed local changes of vorticity with time, and low-level convergence. Indeed, development is defined as low-level convergence in Sutcliffe's [6] and Petterssen's [1] approaches to the problem. Since most forecasters think of development chiefly in terms of increase in circulation and lowering of the central pressure of the Low, a brief description of this aspect of the storm will be presented here. Circulation was measured from the sea level charts using the technique outlined in Petterssen, Dunn, and Means [2]. Results are given in table 3.

These data show that the decrease in central pressure was more or less proportional to the increase in the circulation.

Two periods of intensification are evident. The first corresponds to the initial consolidation of a definite low center late on the 13th from a relatively unorganized trough along the lee slopes of the Rockies. Some vorticity advection was occurring at 300 mb. in the vicinity of the 1000-mb. low center during its initial development. It is believed that the southward movement of a cold High along the slopes of the Rockies also contributed to some increase in the computed circulation about the newly organized Low at this time.

The second period of development was that from 1830 GMT on the 14th to 0630 GMT on the 15th, which corresponds to the time period during which the vorticity advection increased considerably in the middle and upper troposphere in the vicinity of the low center, the Laplacian of the thermal advection also became a significant contributing factor but to a lesser degree in the center of the Low by comparison with the vorticity advection, the level of nondivergence lowered, and precipitation became more general in the vicinity of the low center.

It is noted that during each of the two periods the circulation increased and the central pressure decreased. Later in the North Atlantic this storm deepened considerably more. The central pressure was 974 mb. off Newfoundland at 1230 GMT on the 16th and 945 mb. as it combined with another deep Low southeast of Greenland at 1230 GMT on the 17th.

6. CONCLUSIONS

Some conclusions may be stated with reference to this specific example. These statements have gained further support from other storms studied in this series and from our experience in forecasting development that was derived from the experiment described by Petterssen, Dunn, and Means [2].

(1) Vertical velocities, which in this case were computed only for the central column of the Low, were consistent with the development of precipitation at the low center and with the development of the Low in general. Lowering of the level of nondivergence from above the 300-mb. level to about 600 mb. occurred during the period of strongest intensification, and this appears to be typical of cyclone development.

(2) Patterns of observed local vorticity changes at the 1000-mb. level are in general qualitative agreement with vorticity changes computed from the vorticity and thermal advection terms of equation (2.6) (Appendix). Quantitatively, the computed values are considerably exaggerated. The multiple correlation coefficient was .74. The vorticity advection factor was larger than the Laplacian of the thermal advection factor over the center of the Low, while the thermal advection factor was greater ahead of and to the rear of the moving Low.

(3) Low-level convergence patterns computed from the vorticity advection and thermal advection terms were in general qualitatively consistent with precipitation patterns. Precipitation amounts tended to be greater along the southern edge of the convergence patterns where more warm moist air was present in the lower layers, which emphasizes the importance of moisture content to the air and probably also its degree of stability.

(4) The development discussed in this report is consistent with the simplified hypothesis: "The establishment of a region of appreciable low-level convergence results when and where an area of appreciable vorticity advection in the middle and upper troposphere becomes superimposed upon a low-level frontal system."

(5) The joining of two areas of vorticity advection appeared to contribute to the initial formation.

(6) Patterns of a simple parameter, the Laplacian of the thermal advection for the layer 1000–700 mb., gave a good first approximation to the more elaborately derived

low-level convergence patterns (see par. 3 above), and were, in general, qualitatively consistent with precipitation patterns.

It is the considered opinion of the author that 300-mb. vorticity advection charts and Laplacian of thermal advection charts for the 1000–700-mb. layer (except over high terrain) would be useful guides in the forecasting of initial cyclone development at sea level and of associated precipitation patterns.

ACKNOWLEDGMENTS

The writer wishes to acknowledge his indebtedness to Prof. S. Petterssen, Miss D. L. Bradbury, and Drs. C. W. Newton and M. Estoque for their kind assistance in the preparation of this study.

APPENDIX—SYMBOLS AND EQUATIONS

The equations and symbols used are given below. For a more complete discussion, see Appendix I to Petterssen and Bradbury [3].

1. *Symbols and defining relationships.*—Pressure p is used as vertical coordinate. The vertical velocity ω is expressed by

$$\omega=\frac{dp}{dt} \tag{1.1}$$

where t is time. The equation of continuity is written

$$D=-\frac{\partial\omega}{\partial p} \tag{1.2}$$

where D is horizontal divergence of the wind field represented on pressure contour charts.

The vertical component of the absolute vorticity is replaced by the geostrophic absolute vorticity Q, viz.,

$$Q=\frac{g}{f}\nabla^2 z+f \tag{1.3}$$

where z is the height of an isobaric surface, f is the Coriolis parameter, and ∇^2 is the Laplacian operator on an isobaric surface.

The advection of any quantity x is defined as:

$$A_x=-\mathbf{V}\cdot\nabla x$$

where $\mathbf{V}$ is the velocity vector on an isobaric surface and ∇ is the gradient operator. Thus, the advection is positive or negative according as the wind is from high to low, or low to high, values of x.

If h is the thickness of an isobaric layer, the thickness advection is written:

$$A_h=-\overline{\mathbf{V}}\cdot\nabla h \tag{1.4}$$

Similarly, the vorticity advection at the level of nondivergence L is written:

$$A_{QL}=-(\mathbf{V}\cdot\nabla Q)_L \tag{1.5}$$

2. *Development.*—Following Sutcliffe [6] the amount of convergence $-D$ is taken as a measure of development. Thus,

$$-D=\frac{1}{Q}\frac{dQ}{dt}=\frac{1}{Q}\left(\frac{\partial Q}{\partial t}+\mathbf{V}\cdot\nabla Q+\omega\frac{\partial Q}{\partial p}\right) \tag{2.1}$$

If $\mathbf{C}$ is the velocity with which a motion system (e. g., a cyclone) moves, and $\delta Q/\delta t$ is the local rate of change of Q at a point that retains its position relative to the moving system, we have:

$$\frac{\delta Q}{\delta t}=\frac{\partial Q}{\partial t}+\mathbf{C}\cdot\nabla Q \tag{2.2}$$

and

$$-D=\frac{1}{Q}\left(\frac{\delta Q}{\delta t}+(\mathbf{V}-\mathbf{C})\cdot\nabla Q+\omega\frac{\partial Q}{\partial p}\right) \tag{2.3}$$

At sea level (or 1000 mb.) $\omega=0$

and

$$-D=\frac{1}{Q}\left(\frac{\delta Q}{\delta t}+(\mathbf{V}-\mathbf{C})\cdot\nabla Q\right) \tag{2.4}$$

Thus, except at the vorticity center, this convergence contributes to intensification $\delta Q/\delta t$ as well as motion.

With the symbols defined above, the development equation for sea level (see Petterssen [1]) is written:

$$-D_0Q_0=A_{QL}+\mathbf{V}_0\cdot\nabla Q_0-\frac{g}{f}\nabla^2 A_h-\frac{R}{f}\nabla^2\left[\log\frac{p_0}{p}\left(\overline{\omega(\Gamma_a-\Gamma)}+\frac{1}{c_p}\overline{\frac{dW}{dt}}\right)\right] \tag{2.5}$$

Here, subscript naught refers to sea level (or 1000 mb.) and the bar denotes the mean value from 1000 mb. to this level of nondivergence, $\Gamma_a=dT/dp$, $\Gamma=\partial T/\partial p$, c_p is specific heat at constant pressure, dW/dt is the heat (other than latent) supplied to a unit mass per unit time, T is absolute temperature, and R is the gas constant.

Since

$$\frac{\partial Q_0}{\partial t}+\mathbf{V}_0\cdot\nabla Q_0=-D_0Q_0$$

the foregoing equation may be written:

$$\frac{\partial Q_0}{\partial t}=A_{QL}-\frac{g}{f}\nabla^2 A_h-\frac{R}{f}\nabla^2\left[\log\frac{p_0}{p}\left(\overline{\omega(\Gamma_a-\Gamma)}+\frac{1}{c_p}\overline{\frac{dW}{dt}}\right)\right] \tag{2.6}$$

Since the terms within the brackets cannot be evaluated routinely, an attempt was made to account for the observed vorticity tendency $\frac{\partial Q_0}{\partial t}$ at sea level by using the simplified equation

$$\frac{\partial Q_0}{\partial t}=A_{QL}-\frac{g}{f}\nabla^2 A_h \tag{2.7}$$

3. *Vertical Velocity.*—This was computed only over the sea level center. From equations (2.2) and (2.3), one obtains:

$$\frac{\frac{\delta Q}{\delta t}+\mathbf{V}\cdot\nabla Q-\mathbf{C}\cdot\nabla Q}{Q^2}=\frac{\partial}{\partial p}\left(\frac{\omega}{Q}\right) \tag{3.1}$$

or

$$\int_{p_1}^{p_0}\frac{\frac{\partial Q}{\partial t}+\mathbf{V}\cdot\nabla Q-\mathbf{C}\cdot\nabla Q}{Q^2}\delta p=\left(\frac{\omega}{Q}\right)_0-\left(\frac{\omega}{Q}\right)_1 \tag{3.2}$$

Using the boundary condition that ω_0 vanishes at sea level ($p_0=1000$ mb.) the vertical velocity was obtained for any higher level by columnar interpolation.

For a discussion of accuracy of such computations and the difficulties encountered, reference is made to Petterssen and Bradbury [3].

REFERENCES

1. S. Petterssen, "A General Survey of Factors Influencing Development at Sea Level," *Journal of Meteorology*, vol. 12, No. 1, Feb. 1955, pp. 36–42. (See also *Technical Report* No. 3, Dept. of Meteorology, University of Chicago, 1954.)
2. S. Petterssen, G. E. Dunn, and L. L. Means, "Report of an Experiment in Forecasting of Cyclone Development," *Journal of Meteorology*, vol. 12, No. 1, Feb. 1955, pp. 58–67. (See also *Technical Report* No. 4, Dept. of Meteorology, University of Chicago, 1954.)
3. S. Petterssen and D. L. Bradbury, "An Investigation of Cyclone Development, Storm No. 1," *Technical Report* No. 5, Dept. of Meteorology, University of Chicago, 1954.
4. S. Petterssen, "On the Propagation and Growth of Jet-Stream Waves," *Quarterly Journal of the Royal Meteorological Society*, London, vol. 78, No. 337, July 1952, pp. 337–353.
5. J. F. Appleby, "Trajectory Method of Making Short-Range Forecasts of Differential Temperature Advection, Instability, and Moisture," *Monthly Weather Review*, vol. 82, No. 11, Nov. 1954, pp. 320–334.
6. R. C. Sutcliffe, "A Contribution to the Problem of Development," *Quarterly Journal of the Royal Meteorological Society*, London, vol. 73, No. 317–318, July/Oct. 1947, pp. 370–383.